

MTH 111, Math for Architects, Exam I Spring 2013

Ayman Badawi

QUESTION 1. a) Find an equation of the plane that contains the point (1, 2, 0) and the two vectors : $2i+j-k$, $-3i+2j+5k$.

b) Can we draw the line: $x = 1 + 6t, y = 2 + 3t, z = -3k$ inside the plane in (a)? EXPLAIN CLEARLY.

c) Given a parametric equations of two lines L_1 and L_2, where $L_1 : x = 1 + t, y = 4 - 2t, z = 2 + t$ and $L_2 : x = -2s, y = 9 + s, z = 5 - 6s$. Is L_1 perpendicular to L_2? If the two lines intersect, then find the intersection point.

QUESTION 2. a) Given the plane $P: 3x + y - z = 14$ and $Q = (1, 7, 7)$ not on the plane P. Find the distance between Q and P.

b) Given the line $L_1: x = 3 + 2t, y = 4 + t, z = 2 - 3t$ and $Q = (5, -3, 7)$ not on the line L_1. Find the distance between Q and L_1.

c) Given $V = i - 2j - 2k$. Find two vectors W, F such that W and F are parallel to V , $W \neq F$ but $|W| = |F| = 8.6$

d) Given $V = 6i - 6j + 3k$ and $W = 2i - j + 2k$. Find $Proj_V^W$ and $|Proj_V^W|$. If θ is the angle between V and W what is $cos(\theta)$?

QUESTION 3. a)Find the area of the triangle that has vertices: $(1,1,1),(1,2,5),(2,2,7)$

b)Find the vertex, the directrix and the focus of $8x = y^2 - 8y + 48$. Give a rough sketch of the parabola.

c) Find the center, the foci and the constant k of the ellipse $4x^2 + 8x + y^2 + 2y + 1 = 0$. Give a rough sketch of the ellipse.

QUESTION 4. a) Find the center, the foci, the constant k of the hyperbola $x^2 - 4x - 9y^2 - 18y - 14 = 0$. Give a rough sketch of the hyperbola.

b) Find the equation of the hyperbola that has $(4, 6), (4, -2)$ as its foci, and one of its vertices is $(4, 4)$. Give a rough sketch of the hyperbola.

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MTH 111, Math for Architects, EXAM II , Spring 2013

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QUESTION 1. Given the points: $A = (4, 6)$ and $B = (3, 8)$. Find a point C on the line x = 1 so that $|AC| + |CB|$ is minimum. You need to find the coordinates of the point C.

QUESTION 2. Find y' and don't simplify:

a) $y = 3e^{(5x+4)} + ln(5x^2 + e^x + 7) + \frac{10}{x}$

b) $y = 4x(7x^3 + 2e^x)^3 + \sqrt{2x + 7} + 3x^2$

Question 2 continues:) c) $y = \sqrt[3]{7x+1}$

f) $y = ln[\frac{8x^2+7x-9}{(4x+e^{2x})^4}] + 10$

QUESTION 3. a) Find $lim_{x\to -2} \quad \frac{e^{(3x+6)}-1}{3x^3-12x}$

b) Find $Lim_{x\to 5} \quad \frac{\sqrt{3x+1}-4}{7x-35}$

c) $Lim_{x\to 0} \quad \frac{3x^2}{e^x-x-1}$

QUESTION 4. Let $f(x) = 4e^{2x-6} + 3\sqrt{x-2} + ln(3x-8) - 1$

a) Find the equation of the tangent line to the curve of $f(x)$ when $x = 3$.

b) Use (a) to approximate y when $x = 2.7$.

QUESTION 5. a)Given $e^x + ln(2x + 3y - 8) + yx + 3y - 10 = 0$. Find the equation of the tangent line to the curve at $(0, 3)$.

b) Approximate the y-value when $x = 0.3$

QUESTION 6. a) Find the absolute maximum value of y and the absolute minimum value of y for $f(x) = -xe^{2x^2} + e^{2x^2} + 1$ defined on $[-1, 1]$.

b)For what values of x does $f(x)$ increase? and for what values of x does $f(x)$ decrease?

c) [if you like, it is only worth 2 points, i.e. if you do not feel like doing it, you only lose 2 points] Use only the concept of the first derivative and sketch a rough graph of $f(x)$

QUESTION 7. We want to construct a rectangle with maximum area such that two vertices on the line y = 4 and the other two vertices on the curve $y = 31 - x^2$. What should be the length and the width of such rectangle?

QUESTION 8. Evaluate the following integrals:

a) $\int 7e^{x+1} + \sqrt{x+1} + 4x \, dx$

a/2) $\int \frac{x+2}{x^2+4x+3} \, dx$

a/3) $\int \frac{x^5+x^6-12}{x^7} \, dx$

a/4) $\int (8e^x + 4)(4e^x + 2x + 4)^7 \, dx$

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MTH 111, Math for Architects, Final Exam, Spring 2013

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(There are 20 items, each item = 5 points, total = 100)

QUESTION 1. a) Find an equation of the plane that contains the point $Q = (1, 2, 0)$ and the line L that has parametric equations $L : x = 1 + 3t, y = 5 + t, z = 1 + 4t$

b) Find the distance between Q and L.

c) Choose any two points on the line L, say Q_1, Q_2. Find the area of the triangle Q_1Q_2Q.

QUESTION 2. a) Given $V = i + 2j + 2k$. Find a vector F that is parallel to V such that $|F| = 7.25$

b) Given $V = 3i - 4j$ and $W = 2i + 2j + k$. Find $Proj_W^V$ and $|Proj_W^V|$. If θ is the angle between V and W what is $cos(\theta)$?

c) A particle moves on the ellipse $2x^2 + 5y^2 + 4x + 10y = 70$. The y is decreasing at rate 0.5 cm/sec. Find the rate of change of x at the point (3, 2).

d) Find the vertex, the directrix and the focus for $9y = x^2 - 10x - 11$ and then sketch .

QUESTION 3. a)Given that an ellipse is centered at (2, 4), it has constant $k = 10$ and one of the foci is $(5, 4)$. Write down the equation of the ellipse and then sketch the ellipse.

b) Find the equation of the hyperbola that has $(6, 4), (-2, 4)$ as its foci, and one of its vertices is $(4, 4)$.

QUESTION 4. a) Given the points: $A = (2, 3)$ and $B = (6, 6)$. Find a point C on the line $y = 2$ so that $|AC| + |CB|$ is minimum. You need to find the coordinates of the point C.

b) Find the absolute maximum value of y and the absolute minimum value of y for $y = (x^2 - 3x + 1)e^x$ defined on $[-2, 2]$ (i.e., $-2 \leq x \leq 2$)

c) Find two numbers x, y where $x + 4y = 20$ and xy is maximum. SHOW THE WORK

QUESTION 5. a) Find $lim_{x\to 5}\ \frac{\sqrt{3x-6}-3}{x^2+x-30}$

b) $Lim_{x\to -2}\ \frac{ln(3x+7)}{e^{(x+2)}-2x-5}$

c) Let $f(x) = e^{2x-3} + 2\sqrt{8x-8} + ln(6x-8) + 4$. Find the equation of the tangent line to the curve of $f(x)$ when $x = 1.5$.

QUESTION 6. a)Given $xe^{y-3} + ln(y + x - 4) + yx + y + x - 13 = 0$; also given $(2, 3)$ lies on the curve. You have been asked to approximate the y value when $x = 1.6$, what will you do? SHOW ALL THE WORK AND APPROXIMATE the y value when $x = 1.6$.

b) We want to construct a rectangle with maximum area inside the ellipse $y^2 + 4x^2 = 20$ such that two vertices on the x-axis and the other two vertices on the upper half of the ellipse. What should be the length and the width of such rectangle? SHOW ALL THE WORK.

QUESTION 7. Evaluate the following integrals:

a) $\int 7e^{x+1} + \sqrt{x+1} + 4x^e \, dx$

a/2) $\int \frac{6x^2+18}{x^3+9x+3} \, dx$

a/3) Find the area of the region that is bounded by $f(x) = -x^2 + 3x + 5$ and the line $y = x + 2$ where $0 \leq x \leq 4$.

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